Qualifying Exam

Stellar Astronomy

May 2024

1 The Sun

It is believed that the Sun has been shining for about 5 billion years, and that the longevity of the Sun is due to the way energy is generated at the core.

a. Provide one piece of evidence to indicate the age of the Sun. (5 points)

b. What is the rough temperature of the surface of the Sun. How is this known? What is the temperature at the core? How is this known? (5 points)

c. Describe the processes to produce the energy at the Sun's core. Provide two observations as evidence of such processes. (5 points)

d. It is expected that the Sun will remain stable for another 5 to 7 billion years. Is there evidence for this? (5 points)

e. Describe what happens to the Sun after it can no longer generate energy at the core. (5 points)

2 Processes and structure inside a star

Write down the equations governing the processes and structure inside a star that describe

a. the hydrostatic equilibrium, (5 points)

b. the energy generation, (5 points)

c. and the radiative energy transport. (5 points)

d. Why does the Hertzsprung-Russell diagram (HRD), plotting the surface properties, namely the luminosity versus surface temperature (or absolute magnitude versus color) of a star, is used suitably to characterize its structural stability, or evolution. In such a HRD, the majority of stars (90%) form a sequence, i.e., the main sequence. Explain the sequence. (5 points)

e. Draw a schematic HRD to illustrate the main sequence for Pop I stars versus Pop III stars. Justify the difference. (5 points)

State clearly the meaning of each symbol in the equations.

3 Basic mathematics

a. Calculate following. Give a brief description how to do this calculation.

$$\frac{d}{dx}(x^3 \sin x)$$

(1 points)

b. Calculate following. Give a brief description how to do this calculation.

$$\frac{d}{dx}\left(\frac{\log x}{x}\right)$$

(1 points)

c. Calculate following. Give a brief description how to do this calculation.

$$\frac{d}{dx}(\cos^2 x)$$

(1 points)

d. Calculate following. Give a brief description how to do this calculation.

$$\int xe^x dx$$

(1 points)

e. Simplify following.

$$\frac{10^a}{10^b}$$

(1 points)

f. Astronomers often use the small angle approximation. Use Maclaurin series expansion to show following relation when angle θ is small.

$$\sin\theta \sim \theta$$

(1 points)

4 Coronal Mass Ejection

Assume that the velocity of a CME (Coronal Mass Ejection) directed toward Earth is 400 km/sec and that the mass of the CME is 10^{13} kg.

a. Estimate the kinetic energy constained in the CME. Give your answer in J. (4 points)

b. Estimate the transit time for the CME to reach Earth. (4 points)

c. At mid-latitude regions on the surface of the Earth, such as northern Japan, aurora is usually not observed. In mid-May 2024, there was a forecast of occurrence of aurorae and aurorae were actually observed in northern Japan. Briefly explain how astronomers are able to "predict" the occurrence of aurorae in advance? (4 points)

5 40 Eridani system

The 40 Eridani system in the constellation of Eridanus consists of three objects, 40 Eri A, 40 Eri B, and 40 Eri C. For this problem, we focus on latter two objects, 40 Eri B and 40 Eri C.

a. The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is 0.201 arcsec and the true angular extent of the semimajor axis of the reduced mass is 6.89 arcsec. The ratio of the distances of 40 Eri B and 40 Eri C from the centre of mass is $a_B/a_C = 0.37$. Find the mass of 40 Eri B and 40 Eri C in terms of the mass of the Sun. Describe the method of your calculation. (4 points)

b. The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the Sun. Describe your answer. (Hint: The absolute bolometric magnitude of the Sun is 4.74.) (4 points)

c. The effective temperature of 40 Eri B is 16,900 K. Calculate its radius, and compare your answer to the radii of the Sun and Earth. Describe your answer. (4 points)

d. Calculate the average density of 40 Eri B. Which type of astronomical object is 40 Eri B? Why do you think so? (4 points)

6 Rotation period

Suppose that the Sun were to collapse down to the size of a neutron star (10-km radius). Assuming that no mass is lost in the collapse, find the rotation period of the Sun after the collapse. Describe the method of your calculation. (Hint: The rotation period of the Sun is different at different latitude. In average, it is about 27 days.) (4 points)

7 The lower and upper end of the main sequence

a. The lower end of the main sequence occurs near 0.072 $M_\odot$. What is the main reason for a lack of main sequence stars with masses smaller than 0.072 $M_\odot$? Describe your answer. (4 points)

b. The upper end of the main sequence occurs near 85 $M_\odot$. What is the main reason for a lack of main sequence stars with masses larger than 85 $M_\odot$? Describe your answer. (4 points)

c. The effective temperature and luminosity of 0.072 $M_\odot$ main sequence star are $\log_{10} T_{eff} = 3.23$ and $\log_{10}\left(\frac{L}{L_\odot}\right) = -4.3$, respectively. Calculate the radius of 0.072 $M_\odot$ star in terms of solar radius. Describe the method of your calculation. (4 points)

Constants

π	$\pi = 3.14$
astronomical unit	$1\,\mathrm{au} = 1.50 \times 10^{11}\,\mathrm{m}$
parsec	$1\,\mathrm{pc} = 3.09 \times 10^{16}\,\mathrm{m}$
speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.67 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.67 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.65 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$