Graduate Institute of Astronomy, National Central University
PHD QUALIFYING EXAMINATION 2023 — STELLAR ASTROPHYSICS

1. Radiative pressure

For a star of mass M and luminosity L surround by spherical cloud shell who has both small physical and optical thickness. Suppose the cloud is "gray", that is, the absorption coefficient is independent of the wavelength, and the source function is isotropic.

(1) Show that if the cloud can be ejected by the radiation of the star, the mass to luminosity ratio (M/L) for the star must be less than $\kappa/(4\pi Gc)$ where G is gravitational constant, c is the speed of the light and κ is the opacity of the cloud. (5 points)

(2) If the cloud is ejected by the radiation, show that the terminal velocity (i.e. $r \to \infty$)

$$v^2 = \frac{2GM}{R}\left(\frac{\kappa L}{4\pi GMc} - 1\right)$$

if the cloud starts rest a distance R from the center of the star. (5 points)

2. Stellar Structure

A star of radius R has a density distribution $\rho(r) = \rho_c\left[1 - \left(r/R\right)^2\right]$

(1) Write down the total mass of star M as a function of R and ρ_c. (5 points)

(2) Write down the pressure at the center of star P_c as a function of R and ρ_c. (10 points)

3. Exoplanet

Explain (quantitatively, if possible) for the following questions about using the radial velocity technique for detecting an exoplanet (assume that the orbit of the exoplanet is circular).

(1) Please describe this technique and what orbital parameters can you get from the observation data? (5 points)

(2) How to use this technique to constrain the mass of the exoplanet? (5 points)

(3) Why does the radial velocity technique for detecting exoplanets favor "hot Jupiters" (5 points)

4. Black body radiation

The specific intensity of black body radiation can be written as

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

where h is Planck constant, k is Boltzmann constant and c is speed of light.

(1) Please use the equation to prove the Stephen-Boltzmann law. Also, write down Stephen-Boltzmann constant as the function of the fundamental physical constant h, k and c. (5 points)

(*Hint: you may need the Riemann zeta function of 4* $\sum_{n=1}^{\infty} \frac{1}{n^4} = \zeta(4) = \frac{\pi^4}{90}$ *for your calculation*)

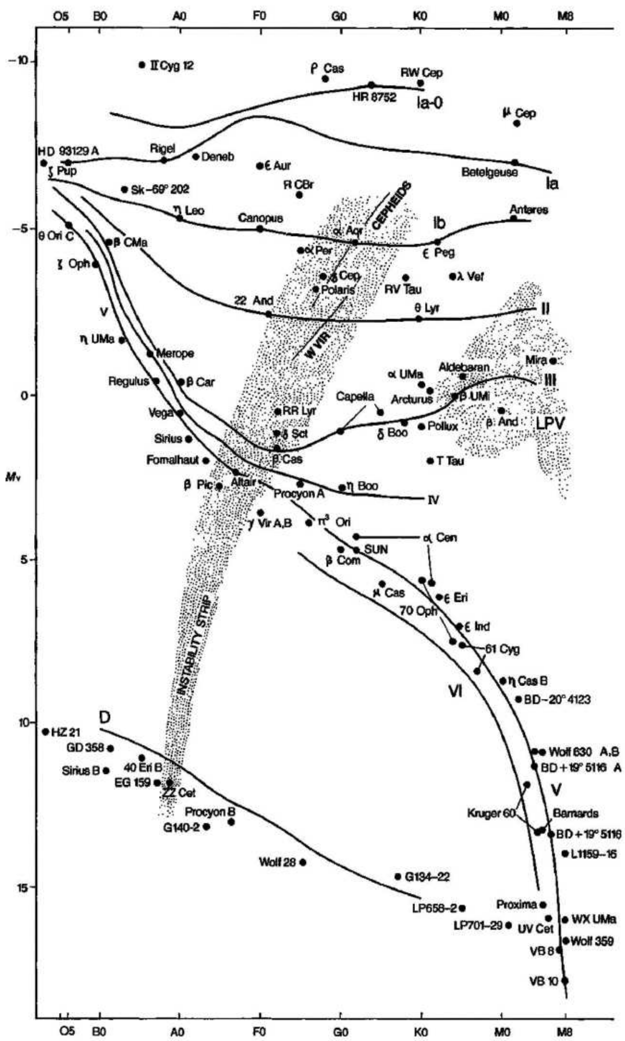
(2) Prove a property of blackbody radiation: monotonicity with temperature, that is, for any given frequency ν if and only if $T_2 > T_1$ then $B_\nu(T_2) > B_\nu(T_1)$. (5 points)

5. Eclipsing binary system

Assume that two stars are in circular orbits about a mutual centre of mass and are separated by a distance a. Assume also that the angle of inclination is i and their stellar radii are r_1 and r_2. Find an expression for the smallest angle of inclination that will just barely produce an eclipse. Describe your answer. (5 points)

6. Distance to a star

The star Pollux is in the constellation of Gemini. Pollux has an apparent visual magnitude of $V \sim 1$. Use following H-R diagram to determine the distance to this star. Describe your answer. (5 points)



7. The most probable speed of gas particles

Show that the most probable speed of the Maxwell-Boltzmann distribution of molecular speeds

$$n_v dv = n \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv$$

is given by

$$v_{mp} = \sqrt{\frac{2kT}{m}}.$$

(10 point)

8. Energy source of stars

1. Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, give a crude estimate how long the Sun could shine at its current rate through chemical processes alone. For simplicity, assume that the Sun is composed entirely of hydrogen. Give your answer in year. Describe your answer. (8 points)

2. Is it possible that the Sun's energy is entirely chemical? Why or why not? (2 points)

9. Temperature of a dust grain around a star

Give a rough estimate of the temperature of a spherical dust grain that is located 100 au from a newly formed F0 main-sequence star. Assume that the dust grain is in thermal equilibrium. Assume also that the dust grain is a perfect blackbody. Typical mass, radius, and effective temperature of F0 main-sequence stars are 1.6 solar mass, 1.4 solar radius, and 7300 K, respectively. Describe your answer. (10 points)

10. Age of a planetary nebula

Consider a planetary nebula with an angular diameter of 15 arcmin that is located approximately 180 pc from Earth.

1. Calculate the diameter of this planetary nebula. Describe your answer. (5 points)

2. Assuming that this planetary nebula is expanding away from the central star at a constant velocity of 20 km per sec, give a rough estimate of its age in year. Describe your answer. (5 points)

Constants

π	$\pi = 3.14$
astronomical unit	$1\,\mathrm{au} = 1.50 \times 10^{11}\,\mathrm{m}$
parsec	$1\,\mathrm{pc} = 3.09 \times 10^{16}\,\mathrm{m}$
speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.67 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.67 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.65 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$