PhD Qualifying Exam (2019)

Stellar Astrophysics

May 2019

1 Thermonuclear Reactions

At the core of the Sun, the central temperature is about 1.5×10^7 K. (a) How is this known? (b) What is the kinetic energy, in unit of keV, of the gas there? (c) The Coulomb electric potential between two nuclei (e.g., protons) is on the order of MeV. Yet two protons can fuse to form a deuterium nucleus, a deuterion, liberating energy. How could the Coulomb barrier be overcome if the average kinetic energy of protons is far insufficient? (d) Given a central density of 150 gcm^{-3}, estimate the ideal gas pressure at the core. (e) This inward pressure must be high, yet it balances the gravitational force, which is zero at the center, to reach a hydrostatic equilibrium. Explain the inconstancy. (f) The Sun at the end of its life is expected to become a 0.6 $M_\odot$ white dwarf supported structurally by electron degenerate pressure. Describe the major differences between the degenerate pressure and the ideal gas thermal pressure. (30 points)

2 Evidence of Stellar Evolution

Our Sun is believed to form out of dense molecular core. First as a protostar, which is observable only in infrared wavelengths or longer, it then became a T Tauri star before reaching the zero-age main sequence as a G-type star with a surface temperature of $\sim$ 5800 K. At an age of roughly 4.6 billion years now, the Sun is expected to continue to shine for another 5 to 7 billion years after which it will turn into a red giant, and eventually the core will contract to become a white dwarf. (a) What is the main energy source of a protostar? (b) Why can't a protostar be seen optically? (c) List 3 lines of observational evidence that a T Tauri star is young. (d) What is the main energy source of a main sequence star? (e) How is the surface temperature of the Sun known? (f) How is the age of the Sun estimated? (g) How is the brightness of the present-day Sun compared to that at the zero-age main sequence? Why? (h) What is the main energy source of a red giant? (i) How does a white dwarf maintain hydrostatic equilibrium in its structure? (j) During which stage did the planetary system form? Why?

(2 points each)

3 Average speed of gas particles

A star consists of huge amount of gas particles. Consider a blob of single-species gas in a box of size $L \times L \times L$. The mass of a gas particle is m, and the temperature of gas is T. Show that root-mean-square speed of gas particles can be written as

$$v_{rms} = \sqrt{\frac{3kT}{m}},$$

where k is the Boltzmann constant. Give assumptions you made, and describe processes of your derivation. (12 points)

4 Binary systems and physical quantities of stars

Physical quantities of stars can be studied by observations of binary systems. Suppose there is an eclipsing binary star system. (a) What is an eclipsing binary? (3 points) (b) The ratio of the effective temperatures of the two stars in an eclipsing binary system can be estimated. Describe the method. Note clearly assumptions you made. (10 points) (c) Radii of two stars can also be measured. Explain the method to estimate radii of two stars in an eclipsing binary system. Mention assumptions you made. (10 points)

5 Adiabatic gas law

The thermodynamics is a powerful tool to investigate stellar structure. The first law of thermodynamics is given by

$$dU = dQ - PdV,$$

where dU is the change in the internal energy of a small mass element, dQ is the amount of heat added to the element, P is the pressure, and dV is the volume change. (a) Now, we consider adiabatic process. What is an adiabatic process? Explain. (3 points) (b) Assume an adiabatic process, and derive the adiabatic gas law

$$PV^{\gamma} = K,$$

where V is the volume of the element, and K is a constant. And, γ is the ratio of specific heats

$$\gamma \equiv \frac{C_P}{C_V},$$

where C_P is the specific heat at constant pressure

$$C_P \equiv \left.\frac{\partial Q}{\partial T}\right|_P,$$

and C_V is the specific heat at constant volume

$$C_V \equiv \left.\frac{\partial Q}{\partial T}\right|_V.$$

Describe clearly the process of your derivation. (12 points)

Constants

Speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^{6}\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$