PhD Qualifying Exam (2016) --- Stellar Astrophysics

1. **Opacity**

   (20%) The opacity refers to the level of difficulty of radiation to travel through a medium. (a) For an incident radiation of intensity I_0 passing a distance x through a gas medium of mass density ρ and of opacity κ_ν, evaluate the emergent intensity. (5 points) (b) What is the Kramers opacity? Describe the functional density and temperature dependence of the Kramers opacity law. (5 points) (c) What is the Rosseland opacity? (4 points) (d) What is the dominant source of opacity in the solar atmosphere ($T_{\rm eff} \approx 6000$ K)? In a brown dwarf ($T_{\rm eff} \approx 1000$ K)? (6 points)

2. **Stellar Mass-Luminosity Relation**

   (10%) There is a mass-luminosity relation for zero-age main sequence stars, in a power-law form, $L \propto M^\alpha$, for which the index α is empirically found to be between 2 and 3 for low-mass ($\mathrm{M}/M_\odot < 0.5$) stars, between 3 and 4 for higher-mass $0.5 < \mathrm{M}/M_\odot < 20$) stars, and is unity for massive stars with Eddington luminosities ($\mathrm{M}/M_\odot > 20$). Assuming hydrostatic equilibrium between ideal gas pressure and gravity, and electron scattering as the dominant opacity in high-mass stars, verify that $\alpha \approx 3$. (10 points)

3. **Gas Pressure and Fermi Energy**

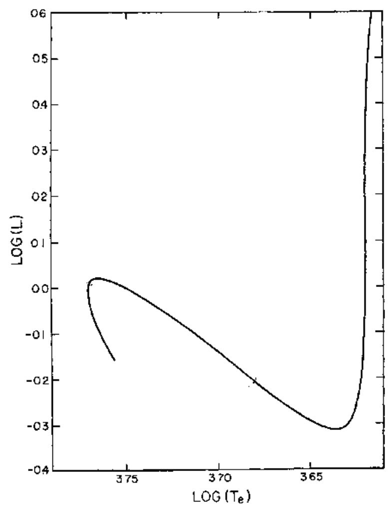
   (10%) It is estimated that presently at the core of the Sun, the temperature is 15 million K and the density is 150 [g cm^{-3}]. Given that the electron degenerate pressure contributes no more than a few percent to the total pressure, the supporting pressure is mainly by ideal gas pressure. (a) Describe how one can estimate --- but do not actually derive --- the electron degenerate pressure. (5 points) (b) The Sun at the end of its life is expected to become a 0.6 $M_\odot$ white dwarf supported structurally by electron degenerate pressure. As a white dwarf, what is its Fermi energy? (5 points)

4. **Binary Stars**

   (15%) Many of the important stellar parameters have been derived from measurements of binary stars. (a) Which physical properties can be measured from observations of a visual binary? Describe the method. (3 points) (b) Important information about stars can also be obtained from eclipsing binaries. Show that the ratio of the effective temperatures of the two stars can be estimated from the observations of a eclipsing binary system. (12 points)

5. **Pre-Main Sequence Hayashi Track**

   (10%) During the pre-main sequence phase, the Sun is supposed to follow a nearly vertical track on the Hertzsprung-Russell diagram, i.e., with almost a constant effective temperature (≈ 4200 K), before the hydrogen nuclear fusion took place in the core (see the figure on the right, modified from Iben, 1965, ApJ, **141**, 993). During this descent, called the Hayashi track, the star is fully convective. (a) Explain why the effective temperature remains the same during this evolutionary phase. Why is the star convective? (5 points) (b) Estimate the timescale of the Hayashi track for the Sun. (5 points)



6. Structure of a star

(15%) The structure of a star can be investigated using physical laws with some assumptions.
(1) Considering a star with an atmosphere in hydrostatic equilibrium, derive the pressure distribution of a star,

$$\frac{dP(r)}{dr} = -G\,\frac{M(r)\,\rho(r)}{r^2},$$

where $P(r)$ is the pressure at the distance r from the center of the star, $M(r)$ is the mass inside the radius r, $\rho(r)$ is the density of the gas at r, and G is the gravitational constant, respectively. (10 points)

Using the equation above, estimate the pressure at the center of the Sun. The mean density of the Sun is 1.41 [g cm^{-3}]. (5 points)

7. Velocities of gas particles

(20%) Velocities of gas particles are described by the Maxwell-Boltzmann distribution. The number of gas particle having a velocity between v and $v + dv$ is expressed as

$$n_v dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2\, dv,$$

where n is the total number density, m is the mass of the gas particle, T is the temperature of the gas, and k is the Boltzmann constant.

(1) Show that the most probable speed of the particles v_{mp} is given by

$$v_{\mathrm{mp}} = \sqrt{\frac{2kT}{m}}$$

(10 points)

(2) The root-mean-square speed of particles v_{rms} can be calculated from the square root of the mean value of v^2. Prove that

$$v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}}$$

Note that the Gaussian integral is $\int_{-\infty}^{+\infty} \exp(-x^2)\,dx = \sqrt{\pi}$.
(10 points)

Constants

Speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^{6}\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$