Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — GALACTIC AND EXTRAGALACTIC ASTROPHYSICS

9:00–13:00, 19th May, 2015

(1) (10 points) **Big Bang theory**
List and describe (at least three) observational evidences that support the Big Bang theory.

(2) (25 points) **Evolution of the universe**
(a) (5 points) For a sphere with mass M and radius r, where the sphere is expanding (or contracting). Show that for a test mass at the edge of sphere the gravitational acceleration can be expressed as:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = \frac{GM}{r(t)} + k\,,$$

where k is a constant.
(b) (5 points) Using $r(t) = a(t)r_0$, where $a(t)$ is the scale factor, show that

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho(t) - \frac{K}{a^2}\,,$$

where $\dot{a} = \mathrm{d}a/\mathrm{d}t$ and $K = -2k/r_0^2$. This is called the Friedmann equation.
(c) (5 points) Based on the result from (b), derive the expression of the critical density of the Universe. What happen to the Universe if the average density of the Universe is greater or smaller than the critical density?
(d) (4 points) If a universe consists of just one component such that $\rho(t) \propto a(t)^{-2}$. Express the Friedmann equation in terms of the scale factor $a(t)$, the Hubble constant $H_0 = \dot{a}(t_0)/a(t_0)$, and the dimensionless energy density $\Omega = \rho/\rho_{\mathrm{critical}}$.
(e) (3 points) What is the time evolution of this universe?
(f) (3 points) Is this universe accelerating or decelerating? Show how you can draw such a conclusion.

(3) (20 points) **Spiral galaxies**
(a) (5 points) What is the typical rotation curve (velocity verse radius) of a spiral galaxy?
(b) (5 points) How do you explain this type of rotation curve and what is its implication?
(c) (5 points) Why cannot spiral arms of galaxies be material arms?
(d) (5 points) How do you explain the existence of spiral arms?

(4) (20 points) **Mass**
(a) (5 points) Astronomers usually measure the mass of molecular gas using CO rotational transitions. However, the CO lines are usually optical thick. How can astronomers measure the molecular mass using an optical thick line?
(b) (5 points) How do astronomers measure the stellar mass of a galaxies?
(c) (5 points) How do people measure the total mass of spiral galaxies or elliptical galaxies?
(d) (5 points) How can people measure the total mass of galaxy clusters?

(5) (15 points) **Thermal and non-thermal emissions**

(a) (5 points) How can we know a radio source to be thermal or non-thermal purely from observations?

(b) (5 points) What are the emission mechanisms of thermal and non-thermal radio sources?

(c) (5 points) How do we know a non-thermal radio source to be young or old?

(6) (10 points) **Stellar dynamics**

The velocity (or energy) distribution of a stellar system might change with time because of the mutual interactions between stars. A very important time scale of the system is the relaxation time $t_{\rm relax}$.

(a) (2 points) Please give a definition of $t_{\rm relax}$.

(b) (8 points) Please derive the formula for $t_{\rm relax}$ by considering strong stellar encounter.