Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

9:00–13:00, 22nd May, 2014

(1) (15 points) **Stellar properties**

Sirius B, the white dwarf companion of Sirius A, has an observed luminosity 0.03 $L_\odot$, a radius 0.008 $R_\odot$, surface temperature 27,000 K, and mass 1.05 $M_\odot$.

(a) (5 points) Describe how these properties, the mass, temperature, luminosity and radius of Sirius B can be estimated by observations.

(b) (5 points) Assuming a radiative temperature gradient and opacity by electron scattering $\kappa = 0.2$ cm^2 g^{-1}, estimate the central temperature of Sirius B.

(c) (5 points) The mass of Sirius B does not exceed the Chandrasekhar limit, the maximum mass for a white dwarf. Quantitatively explain why such a limit exists and state (do not derive) what this limit is numerically in terms of solar masses.

(2) (13 points) **Radiation**

From the definition of the specific intensity

$$dE = I_\lambda \cos\theta \, dA \, dt \, d\Omega \, d\lambda \,,$$

show that

(a) (4 points) The net flux can be written as

$$F = \oint I_\lambda \cos\theta \, d\Omega \, d\lambda \,.$$

(b) (4 points) The net radiation pressure can be written as

$$P_r = \frac{1}{c} \oint I_\lambda \cos^2\theta \, d\Omega \, d\lambda \,.$$

(c) (5 points) Using the equation of radiative transfer, if the structure of a star is spherically symmetric and the source function is isotropic, show that inside the star

$$\frac{dP_r}{dr} = -\frac{\rho(r)\bar{\kappa}r}{c} F(r) \,,$$

where r is the distance from the center of the star, $\rho(r)$ and $\bar{\kappa}(r)$ are the mass density and opacity, respectively.

(3) (13 points) **Exoplanet**

Explain (quantitatively, if possible) for the following questions about using the radial velocity technique for detecting an exoplanet (you may assume the orbit of the exoplanet is circular).

(a) (5 points) Describe this technique and what orbital parameters can you get from the observation data?

(b) (4 points) How to use this technique to constrain the mass of an exoplanet?

(c) (4 points) Why does the radial velocity technique for detecting exoplanets favor “hot Jupiters”?

(4) (12 points) **Eddington standard model**

Consider a star in which the radiation pressure P_r and gas pressure P_g are both important where

$$P_r = \tfrac{1}{3}aT^4\,, \quad P_g = \frac{\rho k_{\rm B} T}{\mu m_{\rm p}}\,,$$

and total pressure $P = P_r + P_g$. Suppose the ratio of radiation pressure P_r and gas pressure P_g is constant all over the star and β is defined as $P_g = \beta P$.

(a) (6 points) Show that the luminosity of the star can be expressed as

$$L = (1-\beta)L_{\rm Edd}\,,$$

where $L_{\rm Edd}$ is the Eddington luminosity of the star.

(b) (6 points) Show that the polytropic index is 3 for this star.

(5) (12 points) **Homologous collapse**

A spherical static cloud with uniform initial density ρ_0 starts to collapse. Suppose this cloud is in absent of rotation and turbulence. The cloud is optically thin and the gravitational potential energy created by the collapse can be efficiently radiated away, so the temperature of the gas remains constant during the collapse. If the internal pressure gradient is too small to influence the motion appreciably, the equation of motion can be written as

$$\frac{d^2r}{dt^2} = -\frac{GM_r}{r^2}\,,$$

where r is the distance from the center of the cloud and M_r is the mass of the sphere interior to radius r.

(a) (6 points) If we are interested only in the surface that encloses M_r which remains constant during the collapse, show that the rate of change of the radius of the surface can be written as

$$\frac{dr}{dt} = -\left[\frac{8\pi}{3}G\rho_0 r_0\left(\frac{r_0}{r}-1\right)\right]^{1/2}\,,$$

when $t = 0$, $r = r0$ and $dr/dt = 0$.

(b) (6 points) Try to solve the equation above and prove that the free-fall time can be written as

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}\,.$$

(6) (10 points) **Stellar birth function**

One often expresses the stellar birthrate function, $B(M,t)$, in terms of the star formation rate, $\psi(t)$, and the initial mass function, $\xi(M)$, in the form $B(M,t)dM/dt = \psi(t)\xi(M)dM/dt$, where M is the stellar mass and t is the time.

(a) (5 points) Describe the concept of $\xi(M)$ and of $\psi(t)$.

(b) (5 points) Elaborate on our current knowledge on $\xi(M)$ and $\psi(t)$.

(7) (10 points) **Cloud collapse**

(a) (5 points) Derive the minimum mass necessary to cause the spontaneous collapse of a cloud of constant temperature T and a homogeneous density of ρ.

(b) (5 points) How is the result modified if this cloud is surrounded by a larger medium which exerts an external pressure of $P_{\rm ext}$ on the cloud?

(8) (15 points) **Post-main sequence stellar evolution**

The Hersprung-Russell diagram below illustrates schematically the post-main sequence evolutionary track of a low-mass star like the Sun. The star is located at position 1 on the main sequence, when the star maintains core hydrogen burning, and transports energy outward by a radiative core and a convective outer layer. The star then evolves to stage 2, etc.

(a) (10 points) Describe the energy source and critical structural changes at each of the stages through 7.

(b) (5 points) An event, called the helium flash, takes place at point A, after which the star immediately fades in luminosity and rises in surface temperature. Explain what a helium flash is, and why such an event should happen?

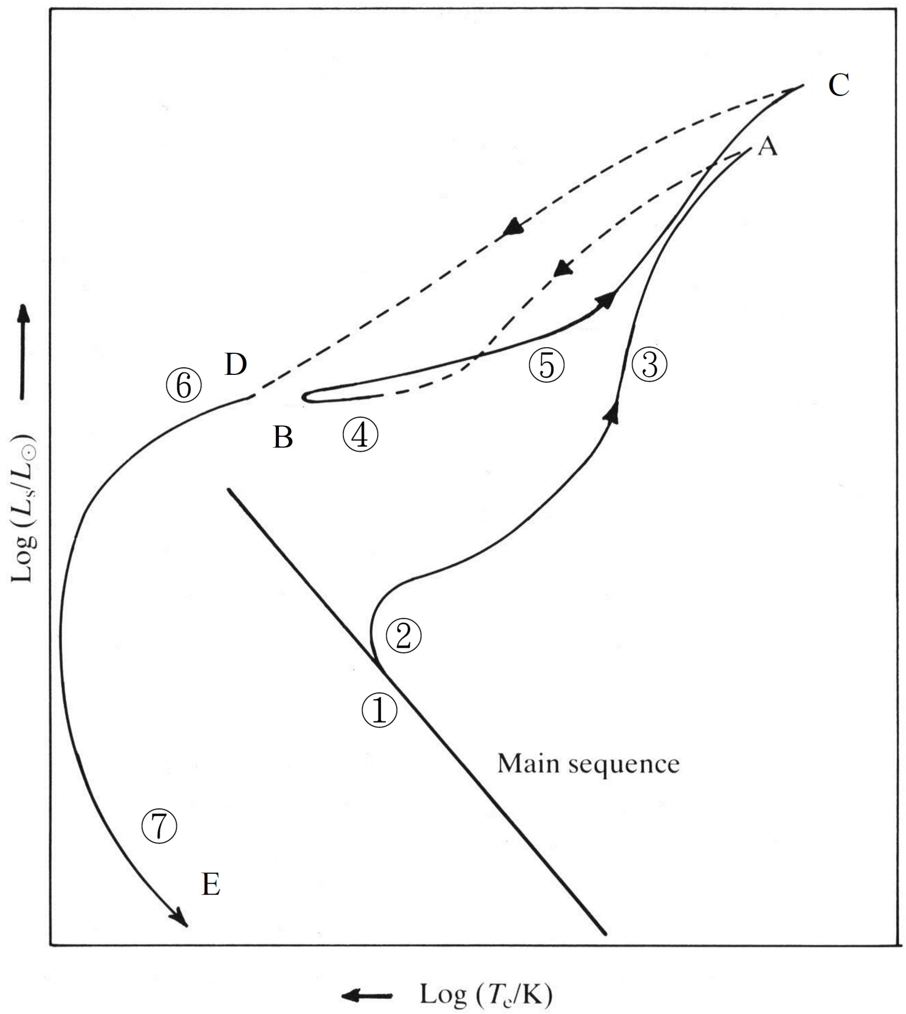

Figure for question (8). Evolutionary track of a Sun-like star in HR diagram.

Constants

Speed of light	$c = 3.00 \times 10^8$ m s^{-1}
Gravitational constant	$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Planck constant	$h = 6.63 \times 10^{-34}$ J s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
Electron volt	$1\,\mathrm{eV} = 1.6 \times 10^{-19}$ J
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J m^{-2} s^{-1} K^{-4}
Radiation constant	$a = 7.56 \times 10^{-16}$ J m^{-3} K^{-4}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Atomic mass unit	$m_H = 1.66 \times 10^{-27}$ kg
electron mass	$m_e = 9.11 \times 10^{-31}$ kg
proton mass	$m_p = 1.6726 \times 10^{-27}$ kg
neutron mass	$m_n = 1.6749 \times 10^{-27}$ kg
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}$ kg
hydrogen atom mass	1.674×10^{-27} kg
helium-3 atom mass	5.009×10^{-27} kg
helium-4 atom mass	6.648×10^{-27} kg
ideal gas constant	$\mathcal{R} = 8.31 \times 10^3$ J kg^{-1} K^{-1}
Solar mass	$M_\odot = 1.99 \times 10^{30}$ kg
Solar radius	$R_\odot = 6.96 \times 10^8$ m
Solar luminosity	$L_\odot = 3.85 \times 10^{26}$J s^{-1}
Mean density of the Earth	$\rho_\oplus = 5.51 \times 10^3$ kg m^{-3}
Earth mass	$M_\oplus = 5.98 \times 10^{24}$ kg
Earth radius	$R_\oplus = 6.38 \times 10^6$ m
Astronomical unit	$1\,AU = 1.50 \times 10^{11}$ m
π	$\pi = 3.14$
cal and J	1 cal = 4.2 J