

# PhD Qualifying Exam (2012) --- Stellar Astrophysics

## Constants

Speed of light	$c = 3.00 \times 10^8 \text{ m s}^{-1}$	helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27} \text{ kg}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	hydrogen atom mass	$1.674 \times 10^{-27} \text{ kg}$
Planck constant	$h = 6.63 \times 10^{-34} \text{ J s}$	helium-3 atom mass	$5.009 \times 10^{-27} \text{ kg}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$	helium-4 atom mass	$6.648 \times 10^{-27} \text{ kg}$
Electron volt	$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$	ideal gas constant	$\mathcal{R} = 8.31 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8} \text{ J m}^{-2} \text{ s}^{-1} \text{ K}^{-4}$	Solar mass	$M_{\odot} = 1.99 \times 10^{30} \text{ kg}$
Radiation constant	$a = 7.56 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$	Solar radius	$R_{\odot} = 6.96 \times 10^8 \text{ m}$
Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$	Solar luminosity	$L_{\odot} = 3.85 \times 10^{26} \text{ J s}^{-1}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27} \text{ kg}$	Earth mass	$M_{\oplus} = 5.98 \times 10^{24} \text{ kg}$
electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg}$	Earth radius	$R_{\oplus} = 6.38 \times 10^6 \text{ m}$
proton mass	$m_p = 1.6726 \times 10^{-27} \text{ kg}$	Astronomical unit	$1 \text{ AU} = 1.50 \times 10^{11} \text{ m}$
neutron mass	$m_n = 1.6749 \times 10^{-27} \text{ kg}$	$\pi$	$\pi = 3.14$
		cal and J	$1 \text{ cal} = 4.2 \text{ J}$

### 1. Cloud Collapse and Star Formation (10%)

An interstellar cloud has a density  $\rho = 2 \times 10^{-21} \text{ km m}^{-3}$ , and a temperature  $T = 10 \text{ K}$ .

- Derive its Jeans mass. How does this compare to the mass of the Sun, and to that of a typical Galactic star cluster? (4 points)
- Derive the free-fall time scale for the cloud to collapse. (4 points)
- The temperature of the cloud core rises when it becomes optically thick. Eventually nuclear reactions are “ignited”, that is, reactions take place effectively when a threshold temperature is reached. Explain why there is a narrow range, or a threshold, temperature (the Gamow peak) within which nuclear reaction occur effectively. (2%)

### 2. Main-Sequence Lifetime (25%)

- There are three ways to transport energy, conduction, convection, and radiation. Which one is the most efficient to transport energy, respectively, in vacuum, in fluid, and in solid? (5 points)
- The Sun is believed to have a main-sequence lifetime of 10 billion years. Consider two stars, one with the mass of 0.1 solar mass and the other of 30 solar mass. During the main-sequence phase, the 0.1 solar-mass star shall use up all the hydrogen in the star, whereas the 30 solar-mass star uses only about 10 percent of hydrogen it originally had. Explain why there is such a difference. (5 points)
- Assuming the mass-luminosity relation of the form  $L \propto M^{3.5}$ , where  $L$  is the luminosity of the star, and  $M$  is the mass of the star, estimate respectively the luminosity of the 0.1 solar-mass star, and of the 30 solar-mass star. (6 points)
- Calculate respectively the main-sequence lifetime of the 0.1 solar-mass star, and of the 30 solar-mass star. (6 points)
- At the end of the main-sequence phase of the 0.1 solar-mass star, all the hydrogen it originally had should have been converted into helium. Are we able to find such a pure helium star? Explain why or why not? (3 points)

### Evolution of the Sun (25%)

- (a) In the HR diagram, plot the evolutionary sequence of the Sun, starting from the pre-main sequence contraction, main-sequence, to the red giant branch, horizontal branch, and asymptotic branch phases. In each phase, state the main energy source and energy transportation mechanism inside the star. (10 points)
- (b) The Sun is thought to have lived for about 5 billion years. How is this inferred? The central temperature of the Sun is believed to be about 15 million K. How is this inferred? The Sun has a “surface” temperature of about 5800 K. How is this inferred? (3 points)
- (c) The Sun is an ensemble of gas with a continuous radial density distribution. Why, then, does the Sun appear to have a “surface” boundary? (2 points)
- (d) After the Sun leaves the main-sequence phase, it evolves to become a subgiant. It is believed that stars with masses between 0.5 and 2.25 solar undergo runaway fusion of helium at their core, i.e., the “helium flash” takes place. Explain the phenomenon, and why such helium flash happens only for stars within this limited mass range? (5 points)
- (e) The Sun will eventually become a white dwarf. Write down the equation of state of a white dwarf. What is the energy source of a white dwarf? (5 points)

### 3. Local Thermodynamic Equilibrium (25%)

- (a) Describe the mean free path. (5 points)
- (b) Describe the local thermodynamic equilibrium (LTE). (5 points)
- (c) The temperature scale height  $H$  is defined as  $H(r) \equiv T(r)/(|dT(r)/dr|)$ , where  $r$  is the distance from the center of the star, and  $T(r)$  is the temperature at  $r$ . The temperature in the solar photosphere changes from 5765 K to 5890 K over a distance of 13.5 km on the vertical direction. What is the temperature scale height of the solar photosphere? (3 points)
- (d) We assume that two hydrogen atoms collide with each other when the distance between the center of two hydrogen atoms is smaller than twice the Bohr radius ( $a_{\text{Bohr}} = 0.529 \text{ \AA}$ ). What is the collisional cross section of a hydrogen atom? (3 points)
- (e) Assuming a pure, neutral hydrogen photosphere for the Sun, with a mean density  $\rho = 2.5 \times 10^{-4} \text{ kg m}^{-3}$ , calculate the mean free path in the solar photosphere. (5 points)
- (f) Considering the mean free path and temperature scale height of the solar photosphere, is the solar photosphere considered to satisfy the LTE condition? (4 points)

### 4. General Astrophysical Concepts (15%)

Please briefly explain the following terms: (a) Schönberg-Chandrasekhar limit, (b) Chandrasekhar limit (of a white dwarf), (c) Eddington limit, (d) Roche limit, (e) Diffraction limit (of a telescope). (3 points each)