Qualifying Exam

— Stellar Astrophysics —

May 2011

- Print clearly.
- Answer either in English or in Chinese.
- Note clearly the meaning of each symbol.
- Show assumptions you made.

1 Spectral lines

At 500.7 nm, there is a doublet forbidden spectral line of [O III]. The line is observed in H II regions and planetary nebulae to be much stronger than the neighboring Hβ line at 486.1 nm.

1. What does O III mean?
2. Explain what a forbidden spectral line is.
3. What is an H II region?
4. Explain why the [O III] line is very prominent in terms of the collisional rate and stellar radiation field.

(8 points)

2 Hayashi track

The "Hayashi track" refers to an evolutionary sequence of a star in its pre-main sequence phase on the Hersprung-Russel diagram.

1. Draw the Hayashi track for the Sun (you should roughly label the axes), and give a physical explanation of the shape of the track.
2. How does the Hayashi track of a 1 $M_\odot$ star compare with that of a 0.5 $M_\odot$ star, and to that of a 5 $M_\odot$ star? Why? How about a 20 $M_\odot$ star?

(10 points)

3 Collapse of gas

Consider a dark cloud globule of mass M, radius R and temperature T, experiencing an external pressure P_0.

1. Derive the Jeans criterion under which the globule will collapse under self-gravity.
2. Derive the time scale for such a collapse process.

(20 points)

4 Stellar evolution

1. On an HR diagram, plot the evolutionary sequence of the Sun, starting from the collapse of a cloud, the pre-main sequence, main-sequence, to the red giant branch, horizontal branch, and asymptotic giant branch phases.

2. On each phase, explain the main energy source and energy transportation mechanism inside the star.

3. The Sun is thought to have lived for about 5 billion years. How is this inferred? The central temperature of the Sun is believed to be 15 million K. How is this inferred?

4. The Sun is expected to end its life as a white dwarf. Write down the equation of state of a white dwarf. What is the energy source of a white dwarf?

(12 points)

5 Radius of a star

1. The adaptive optics (AO) offers high spatial resolution imaging. AO imaging at K-band ($\lambda \sim 2.2$ μm) using 8.2-m telescope is able to achieve diffraction limited imaging. Calculate the expected angular resolution of AO imaging at K-band using 8.2-m telescope. (5 points)

2. For majority of stars, we are not able to measure the stellar radius by direct imaging. Calculate the angular diameter of a star with the radius $R = 10R_\odot$ at 10 pc away from the Sun. (5 points)

3. Although many stars cannot be resolved by direct imaging, we now know the stellar radii for many stars. Explain how we can estimate the size of a nearby star. Note that there are 3 observable quantities which are relatively easy to obtain for nearby stars. Those are (1) apparant magnitude, (2) spectrum, and (3) annual parallax. (5 points)

4. Consider a white dwarf of the luminosity $L = 10^{-2}L_\odot$ and $T_{eff} = 20000$ K. What is the size of this white dwarf? (5 points)

6 Binary systems

Many of stellar properties have been obtained from studies on binary systems.

1. By measuring the orbital motion, we are able to estimate the total mass of the binary system. Consider the balance between the gravity and centrifugal force for both the primary and secondary, and derive the relationship

$$M_1 + M_2 = \frac{4\pi^2}{G}\frac{a^3}{P^2}. \tag{1}$$

   Here, M_1 is the mass of the primary, M_2 is the mass of the secondary, a is the semimajor axis, and P is the orbital period. Note that the semimajor axis a can be written as $a = a_1 + a_2$, here a_1 is the distance between the center of the mass and the primary and a_2 is the distance between the center of the mass and the secondary. (5 points)

2. Sirius is the brightest star as seen from the Earth except the Sun (and some planets and the Moon). Sirius is known to be a visual binary with the semimajor axis $a = 7.56$ arcsec on the sky and the orbital period $P = 50.09$ yr. The annual parallax of Sirius is measured to be 0.379 arcsec. Calculate the total mass of Sirius system. (5 points)

3. What is the spectroscopic binary? (5 points)

4. For spectroscopic binaries, we are able to measure the periodic changes of the radial velocity of spectral lines. The amplitudes of radial velocity changes can be written as

$$K_1 = \frac{2\pi a_1}{P}\sin i, \tag{2}$$

$$K_2 = \frac{2\pi a_2}{P}\sin i. \tag{3}$$

Here, K_1 is the amplitude of radial velocity change for the primary, and K_2 is the amplitude of radial velocity change for the secondary. i is the inclination angle of the orbital plane. Consider the balance between the gravity and centrifugal force, and show the relationship

$$\frac{M_2}{M_1} = \frac{K_1}{K_2}. \tag{4}$$

The ratio of masses can be calculated, if both K_1 and K_2 are known. (5 points)

5. For some spectroscopic binaries, lines of the secondary is too faint to be detected. Even for these cases, we are able to obtain a constraints on masses. Use following 3 equations

   - balance of gravity and centrifugal force for the primary,
   - Kepler's 3rd law for the binary system,
   - amplitude of radial velocity change for the primary (eq. 2)

   and derive

   $$f(M) = \frac{PK_1^3}{2\pi G}. \tag{5}$$

   Here, $f(M)$ is the mass function

   $$f(M) \equiv \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2}. \tag{6}$$

   If the mass of the primary is known, then we are able to set a constraint on the secondary mass. (5 points)

6. Recently, many exoplanets are detected by the radial velocity method. We can set the constraint on the mass of the exoplanet. Now, we calculate the lower limit of the mass of the planet around 51 Pegasi. According to the IAU Circular No. 6251, the orbital period of the planet is $P = 4.2293 \pm 0.0011$ days, and the amplitude of the radial velocity change of the host star 51 Pegasi is $K_1 = 0.059 \pm 0.003$ km/sec. Calculate the lower limit of the mass of the exoplanet around 51 Pegasi ($M_2 \sin i$). Note that $M_1 \gg M_2$ for this case. Compare the result with the Jupiter mass ($M_J = 1.8986 \times 10^{27}$ kg). (5 points)

Constants

Speed of light	$c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^8\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^6\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$