PhD Qualifying Exam (2010) --- 星球天文物理

1. What is limb darkening? Can the limb darkening be seen if the star were isothermal, that is, the temperature is a constant all over the star? Why? Finally, explain how to use transit effect of an exoplanets to probe the limb darkening to its host star. (10 points)

2.

(a) Consider radiative transport, prove that the radiative pressure gradient can be written as

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c}F$$

where $\bar{\kappa}$ is the mean opacity, ρ is mass density and F is radiative flux (4 points)

(b) Prove that radiative temperature gradient can be written as

$$\frac{dT(r)}{dr} = -\frac{3}{16\pi ac}\frac{\bar{\kappa}\rho(r)}{T^3(r)}\frac{L(r)}{r^2}$$

where the $L(r)$ is the luminosity at radius r, a is radiation constant and c is speed of light. (4 points)

(c) Show that the star can be stable only if the mass (M) to luminosity (L) ratio

$$\frac{M}{L} > \frac{\bar{\kappa}}{4\pi Gc} \quad \text{(4 points)}$$

(d) From (c), please derive the famous Eddington luminosity

$$L_{edd} = \frac{4\pi GMm_H c}{\sigma_T}$$

where m_H is the hydrogen atom mass and σ_T is the Thomson cross section. (3 points)

3. For a star whose density is $\rho(r) = \rho_c\left[1-(r/R)^a\right]$ where ρ_c is center density,

R is the radius of the star and $a > 0$.

(a) Calculate the mass of the star (5 points)

(b) Calculate the pressure at the center of the star (5 points)

(c) Calculate the pressure inside the star, that is, $P(r)$ for $a = 1$ (5 points)

4.

(a) If the equation of state of gas for a star is $P = K\rho^{\gamma}$, prove that the condition for dynamic stability is $\gamma \geq 4/3$. (4 points)

(b) The equation of state for non-relativistic degeneracy gas can be written as $P = K\rho^{5/3}$. Estimate the mass-radius relation of this degenerate star. (3 points)

(c) As the mass increase, the electrons gradually become ultra-relativistic and the equation of state becomes $P = K'\rho^{4/3}$. Show that there is a limiting mass for this case and estimate it in term of K'. (3 points)

5. (a) The maximum mass of a cloud to maintain hydrostatic equilibrium is called the Jeans mass. For a cloud of uniform temperature *T*, and average density *ρ*, find its Jeans mass. (5 points) (b) As a cloud collapses often it fragments, until a lower mass limit of fragments is reached. Find this minimum mass. (5 points) (c) Assuming that a star of mass *M* has no nuclear energy sources, find the rate of contraction of its radius, if it maintains a constant luminosity *L*. (5 points)

6. There is a finite range of mass for stars to be structurally stable. Explain and derive as quantitatively as possible why there is a maximal and a minimal value, respectively, for stellar masses. (10 points)

7. Compare the main-sequence stars of Population I and Population II stars, in terms of their temperature, density, luminosity, and main-sequence lifetimes. Assume a Kramers opacity $\kappa=\kappa_0 Z\rho^{-3.5}$, and an energy generation rate $q=q_0 Z\rho T^n$. (10 points)

8. (a) Calculate the equilibrium temperature of a planet of radius a_p, mass m_p, at a distance *d* from the Sun, which has a surface temperature of $T_\odot$ and a radius of $R_\odot$. (5 points) (b) Estimate such equilibrium temperature for the Earth, and

compare it to the average surface temperature of the Earth. (5 points) (c) Calculate where the “snow line” is in the solar system. (5 points)