Question 1: The typical velocity and number density of solar wind at the Earth orbit is 500 km/sec and 10 protons/cm^3, respectively. Calculate the mass loss rate of the Sun. Estimate the mass loss for the main-sequence duration, and compare it to the mass of the Sun.

Question 2: The mass of stars can be estimated using the observations of binary systems.

1. For visual binary systems, one can estimate the total mass of the system using the balance of the gravity and centrifugal force. Derive the relationship between the total mass, semimajor axis, and orbital period of the system.

2. Sirius has a companion. The annual parallax of the Sirius is $p = 0.378$ arcsec, and the apparent separation of Sirius and its companion is 7.62 arcsec. The orbital period of this system is 50.1 years. Calculate the total mass of the system, and express it in the unit of solar mass.

3. The mass of spectroscopic binaries are estimated from the radial velocity change. The mass function f(M) is expressed as

$$f(M) = \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = \frac{PK_1^3}{2\pi G}. \quad (1)$$

   Here, M_1 is the mass of the star, M_2 is the mass of the companion, i is the inclination of the companion's orbit, P is the orbital period, and K_1 is the amplitude of the radial velocity change. The existence of a planet around the star 51 Pegasi is known. Constrain the mass of the planet. The orbital period of the planet is $P = 4.2293 \pm 0.0011$ days, and the amplitude of the radial velocity change is $K_1 = 0.059 \pm 0.003$ km/sec. 51 Pegasi is a solar analogue star. Show the mass of the planet in the unit of Jupiter mass ($M_J = 1.899 \times 10^{27} kg$).

Question 3: The excess emission at the infrared wavelength is observed for number of main-sequence stars, and this infrared excess is interpreted as an existence of the debri disk around the central star.

1. Estimate the equilibrium temperature of the dust particles at 100 AU from the central star assuming the luminosity of the star is equal to that of the Sun. Assume the dark surface for the dust particles.

2. Wien's displacement law is expressed as

$$\lambda_{peak} = \frac{2.897 \times 10^-3}{T}. \quad (2)$$

Estimate the peak wavelength of the blackbody radiation of the dust particle at 100 AU from the central star.

Question 4: The dust particles in the debri disk of the main sequence star are affected by Poynting-Robertson drag. Due to the orbital motion of the dust particle, the radiation from the central star is coming from slightly forward direction and the dust particles have deceleration. As a result, dust particles spiral inward to the star. Now we assume the star has the luminosity equal to that of the solar luminosity.

1. The quantity β is defined as the ratio of the radiation pressure and gravitational force on the dust particle.

$$\beta = \frac{F_{rad}}{F_{grav}} \tag{3}$$

   The radiation pressure can be estimated as

$$F_{rad} = \frac{L_\odot A Q_{PR}}{c 4\pi R^2}. \tag{4}$$

   Here, $L_\odot$ is the solar luminosity, A is the geometric cross-section of the particle, R is the distance from the central star to the dust particle, and Q_{PR} is the radiation pressure coefficient. Assume $Q_{PR} = 1$, and express the β as a function of the radius and mean density of the dust particle. (Assume the spherical shape for the dust particle.)

2. The change of the semimajor axis of the dust particle is expressed as

$$\frac{da}{dt} = -\frac{\beta G M_\odot (2 + 3e^2)}{ac(1 - e^2)^{\frac{3}{2}}}. \tag{5}$$

   Estimate the timescale of the orbital change of the dust particle. Assume the circular orbit for the orbital motion of the dust particle.

3. How long time is needed to change the dust particle with the size of 100 micron at 1 AU from the central star? Compare this timescale to the main-sequence duration of the star.

Question 5.

(a) Suppose the Local Thermodynamic Equilibrium assumption holds for the stellar atmosphere. Prove that for the optically thin line, the absorption line depth R_λ can be written as

$$R_\lambda = \frac{2}{3}\frac{\kappa_L}{\kappa_c}\frac{d\ln B_\lambda}{d\tau_c}\bigg|_{\tau_c=2/3}$$

where κ_c is opacity of continuum, $\kappa_L \equiv \kappa_\lambda - \kappa_c$ and κ_λ is the opacity at wavelength λ. For optically thin line, $\kappa_L \ll \kappa_c$.

(b) What type of spectral lines would you expect see if the temperature of a star's atmosphere is increasing outward? Why?

(c) If you are able to measure the equivalent width of the absorption line from a specified transition of an element (in either atomic or ionic state), please explain how to estimate the abundance of the element if the electron pressure in the stellar atmosphere is also known.

Question 6.

Please use the solution of radiative transfer equation and the concepts of quantum mechanics to explain the Kirchhoff's three laws of spectroscopy listed as following:

(i) A hot dense object produces light with a continuous spectrum

(ii) A hot diffuse gas produces emission spectral lines at discrete wavelengths which depend on the energy levels of the atom of the gas

(iii) A hot dense object surround by a cool dilute gas produces light with an almost continuous spectrum which has absorption lines depending on the energy levels of the atoms in the gas.

Question 7.

(a) Consider radiative transport, prove that the radiative pressure gradient can be written as

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c}F$$

where $\bar{\kappa}$ is the mean opacity, ρ is mass density and F is radiative flux

(b) Prove that radiative temperature gradient can be written as

$$\frac{dT(r)}{dr} = -\frac{3}{16\pi ac}\frac{\bar{\kappa}\rho(r)}{T^3(r)}\frac{L(r)}{r^2}$$

where the $L(r)$ is the luminosity at radius r, a is radiation constant and c is speed of light.

(c) Show that the star can be stable only if the mass to luminosity ratio

$$\frac{M}{L} > \frac{\bar{\kappa}}{4\pi Gc}$$

Question 8.

(a) If the equation of state of gas for a star is $P = K\rho^{\gamma}$, prove that the condition for dynamic stability is $\gamma \geq 4/3$.

(b) For the equation of state for non-relativistic degeneracy gas can be written as $P = K\rho^{5/3}$, derive the mass-radius relation of white dwarf.

(c) As the mass increase, the electrons gradually become ultra-relativistic and the equation of state becomes $P = K'\rho^{4/3}$. Show that there is a limiting mass for this case and estimate it in term of K'.