Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

30th May, 2005

Please answer 5 out of the following 7 problems.

(1) (20 points)
In the theory of radiative transfer in stellar atmospheres, it is important to relate the surface intensities $I_\lambda(0,\theta)$ to the source strengths $S_\lambda(\tau,\lambda)$. Please answer the following questions by using a linear approximation of $S_\lambda(\tau,\lambda)$.

(a) (7 points) What is the so-called "limb darkening effect"? Please explain the limb darkening effect by using the relation between $I_\lambda(0,\theta)$ and $S_\lambda(\tau,\lambda)$?

(b) (7 points) Please give three examples of stars with a well-measured limb-darkening effect. How can the limb-darkening effects be measured in these examples? Please explain how the transit effect of the exoplanets can be used to study the limb darkening of host stars.

(c) (6 points) Please derive and describe the physical significance of the Eddington-Barbier relation.

(2) (20 points)
The spectra of stellar atmospheres are determined by several important factors. One of them has to do with the optical depths related to the absorption of the photon fluxes.

(a) (4 points) Please describe precisely the definition of a gray stellar atmosphere and the meaning of the non-grey cases?

(b) (8 points) Please describe in general terms, what the major contributors are to optical opacities in stellar atmospheres and then use the Saha equation to explain how the temperature would affect their relative importance?

(c) (8 points) Please draw a diagram showing the forms of the absorption coefficients κ_λ per gram of hydrogen and the negative hydrogen ion H^-. Please indicate clearly the position of the H Balmer jump. Why the temperature of a hot star can be determined from the Balmer jump?

(3) (20 points)
Please write down the expressions for the equation of state of the following types of gases:

(a) (2 points) an ideal ion gas

(b) (2 points) an ideal electron gas

(c) (2 points) a degenerate electron gas

(d) (2 points) a relativistic degenerate electron gas

(e) (2 points) the radiation pressure in terms of temperature

(f) (10 points) Please use a schematic diagram to illustrate the evolutionary tracks in the central temperature and density of stars of different masses. (You should include stellar masses from $< 0.1M_\odot$ to $> 100M_\odot$.)

(4) (20 points)

(a) (8 points) Please derive the Virial Theorem and from it compute the Jeans mass for a typical interstellar molecular cloud with a temperature $T \sim 50$ K and number density $n \sim 10^3$ cm^{-3}.

(b) (4 points) How is the derivation of the Jeans mass modified if the cloud has a magnetic field with strength B?

(c) (4 points) In general for such a cloud of temperature T and number density n, with an initial radius R, if it begins to collapse under its own gravity, calculate the free-fall time scale.

(d) (4 points) Please estimate the average temperature of a star of mass M and radius R. Evaluate its value for our sun.

(5) (20 points)

(a) (8 points) Derive the mass-radius relation for white dwarfs.

(b) (4 points) What would white dwarfs of different masses be located in the H-R diagram?

(c) (8 points) In the outer layer of a white dwarf, the pressure $P(T) \propto (M/L)^{1/2}T^{17/4}$ (the radiative zero solution), where M is the mass, L the luminosity and T is the temperature. Show that the temperature profile throughout the outer layer of a white dwarf of mass M and radius R is given by

$$T(r) = GM\left(\frac{4\mu m_{\mathrm{H}}}{17k_{\mathrm{B}}}\right)\left(\frac{1}{r} - \frac{1}{R}\right),$$

where the constants have usual meanings.

(6) (20 points)

(a) (5 points) A star, e.g., the Sun, evolves while it is on the main sequence, usually becoming cooler and brighter in comparison to its zero-age main sequence stage. Please explain qualitatively why this is so.

(b) (5 points) The Sun will end its life by blowing off the envelope of the red giant ("planetary nebula"), with the stellar core eventually becoming a white dwarf. Describe the physical principles that lead to the runaway processes of the planetary nebula phenomenon (or, why does a red giant puff?)

(c) (5 points) Why do planetary nebulae show only emission lines in their spectra?

(d) (5 points) What are the formation mechanisms and the energy source of these emission lines?

(7) (20 points)

Explain the following terms. You can simply give a numerical value without elaborative derivation, but should do more than a simple translation.

(a) (4 points) Schonberg-Chandrasekhar limit

(b) (4 points) Chandrasekhar limit

(c) (4 points) Eddington limit

(d) (4 points) Roche limit

(e) (4 points) Rayleigh's resolution limit

$a = 4\sigma/c = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}
$c = 3.00 \times 10^{8}$ m s^{-1}
$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
$h = 6.626 \times 10^{-34}$ J s
$k_{\mathrm{B}} = 1.38 \times 10^{-23}$ J K^{-1}
$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg $= 9.11 \times 10^{-28}$ g
$m_{\mathrm{H}} = 1.67 \times 10^{-27}$ kg $= 1.67 \times 10^{-24}$ g
$N_{\mathrm{A}} = 6.02 \times 10^{23}$ mol^{-1}
eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg
$L_{\odot} = 3.86 \times 10^{26}$ W $= 3.86 \times 10^{33}$ erg s^{-1}
$M_{\odot} = 1.99 \times 10^{30}$ kg $= 1.99 \times 10^{33}$ g
$R_{\odot} = 6.96 \times 10^{8}$ m
$T_{\mathrm{eff}\odot} = 5,780$ K