Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

26th May, 2004

(1) (10 points)
A stellar atmosphere in which the opacity is assumed to be independent of wavelength is called a gray atmosphere. With the Eddington approximation, show that the vertical temperature variation in a plane-parallel atmosphere in LTE is $T^4 = (3/4)T_e^4(\tau + 2/3)$, where T_e is the effective temperature.

(2) (10 points)
Given that chemical potential μ of an ideal classical gas with number density n is given by

$$\mu = mc^2 + k_{\rm B}T \log\left(\frac{nh^3}{g(2\pi m k_{\rm B}T)^{3/2}}\right)$$

where g is the internal degrees of freedom (or multiplicity or degeneracy) of the gas particle. Show that the Saha equation for $\mathrm{H} + \gamma \rightleftharpoons \mathrm{H}^+ + \mathrm{e}^-$ is

$$\frac{n(\mathrm{H}^+)}{n(\mathrm{H})} \approx \frac{(2\pi m_{\rm e} k_{\rm B}T)^{3/2}}{n(\mathrm{e})h^3}\exp(-E_{\rm i}/k_{\rm B}T)$$

where $E_{\rm i} = 13.6$ eV is the first ionization potential of hydrogen. (State clearly the assumptions you made.) Do you expect the correct ionization rate at the centre of the Sun can be obtained from the above equation? If not, will the correct ionization rate be greater or smaller? Why?

(3) (20 points)
Sirius is a visual binary with a period of 49.94 years. Its measured trigonometric parallax is 0.377" and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is 7.62". The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_{\rm A}/a_{\rm B} = 0.466$.

(a) (10 points) Find the mass of each member of the binary system.

(b) (5 points) The absolute bolometric magnitude of Sirius A is 1.33, and Sirius B has an absolute bolometric magnitude of 8.57. Determine their luminosities, in units of solar luminosity.

(c) (5 points) The effective temperature of Sirius B is estimated to be about 27,000 K. Estimate its radius, and compare the results to the radii of the Sun and Earth.

(4) (20 points)
Though currently there are no direct observations of the first-generation stars of the universe, or Population III stars, some data on distant quasars seem to hint on their existence. Based on your knowledge of stellar structure and evolution, describe how the characteristics of the lives of these stars should differ from those of Pop I stars, in terms of cloud collapse, fragmentation processes, the stellar luminosity, effective temperature, chemical composition, thermonuclear processes, etc.

(5) (20 points)

A self-gravitating sphere of mass M has a density distribution $\rho = \rho_c(1 - r/R)$.

(a) (5 points) Find the core pressure in terms of M and ρ_c.

(b) (5 points) Find the mass-radius relation of a white dwarf, in which the core is solely supported by fully degenerate non-relativistic electrons.

(c) (10 points) Suppose the core of a pre-main-sequence star is supported by fully degenerate non-relativistic electrons and classical ions. Sketch the core temperature against ρ_c for different masses M. Hence discuss what will happen at the core when stars of different masses contract (i.e., when ρ_c increases).

(6) (20 points)

A self-gravitating gas sphere obeys a polytropic relation $P = K\rho^\gamma$, where K and γ are constants.

(a) (15 points) For a given mass M, show that its total energy E has a maximum/minimum with respect to the radius R when γ is smaller/larger than $\frac{4}{3}$. Thus write down the condition on γ for a stable sphere, and show that $M \propto \bar{\rho}^{(3\gamma-4)/2}$ for such a sphere, where $\bar{\rho}$ is the average density of the sphere.

(b) (5 points) Observations of main-sequence stars give $R \propto M^{0.7}$. Find the relation between M and $\bar{\rho}$. Is (a) consistent with this observation? If not, why?

$a = 4\sigma/c = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}

$c = 3.00 \times 10^{8}$ m s^{-1}

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$h = 6.626 \times 10^{-34}$ J s

$k_B = 1.38 \times 10^{-23}$ J K^{-1}

$m_e = 9.11 \times 10^{-31}$ kg $= 9.11 \times 10^{-28}$ g

$m_H = 1.67 \times 10^{-27}$ kg $= 1.67 \times 10^{-24}$ g

$N_A = 6.02 \times 10^{23}$ mol^{-1}

eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg

$L_\odot = 3.86 \times 10^{26}$ W $= 3.86 \times 10^{33}$ erg s^{-1}

$M_\odot = 1.99 \times 10^{30}$ kg $= 1.99 \times 10^{33}$ g

$R_\odot = 6.96 \times 10^{8}$ m

$T_{\mathrm{eff}\odot} = 5,780$ K