# Institute of Astronomy, National Central University

## PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

28th May, 2003

(1) (20 points)
A model star of mass $M$ has a density distribution $\rho = \rho_c[1 - (r/R)^a]$, where $a > 0$.

(a) (10 points) Find the radius, core pressure, core luminosity, core energy generation per unit volume of the star (in terms of $M$ and $\rho_c$).

(b) (10 points) Comment on the structure (i.e., density, pressure, luminosity, energy production distribution) of the model star when compare to a more realistic star (e.g., the Sun). What improvements on the model would you suggest?

(2) (20 points)
The denstiy of a star of mass $M$ is constant.

(a) (5 points) If the equation of state of the gas at the core of the star is $P_c \propto \rho_c^\gamma$, find the range of $\gamma$ such that the core of the star is dynamically stable. Give reasons.

(b) (5 points) In the post-main sequence evolution of a low mass star, there is a phase called helium flash, which occurs when the helium core is degenerate. Describe this instability. Can this instability be explained by (a)? If yes, how? If no, why?

(c) (10 points) When a star exhausts its nuclear fuel, its core will start to contract under its own gravity to become a white dwarf. For a white dwarf supported by the degenerate pressure of nonrelativistic electrons, show that the relation between the mass and the radius of the remnant is $R \propto M^{-1/3}$. What determines the central temperature of a white dwarf? The more massive of a white dwarf, the smaller of its size and the higher of its density. If the degeneracy pressure is provided by relativistic electrons, show that there is an upper limit to the mass for hydrostatic equilibrium to maintain. What is the radius of a relativist degenerate white dwarf?

(3) (20 points)
Assume a star has pressure $P \propto \rho^a T^b$, opacity $\kappa \propto \rho^c T^d$ and energy production per unit volume $\epsilon \propto \rho^e T^f$. Find the surface temperature-luminosity relation (i.e., theoretical H-R diagram) of the following stars.

(a) very high mass stars: $a = 0$, $b = 4$, $c = 0$, $d = 0$, $e = 2$, $f = 17$

(b) high mass stars: $a = 1$, $b = 1$, $c = 0$, $d = 0$, $e = 2$, $f = 17$

(c) low mass stars: $a = 1$, $b = 1$, $c = 1$, $d = -7/2$, $e = 2$, $f = 4$

(4) (20 points)

(a) (10 points) The slab of gas (e.g., the atmostphere of a star, HII region) is uniform in density and temperature. It satisfies the radiative transfer equation $\mathrm{d}I_\nu/\mathrm{d}z = -\kappa_\nu I_\nu + j_\nu$, and $\kappa_\nu \propto \rho T^{-7/2} \nu^{-\beta}$. Describe your approach to determine the density and temperature of the slab of gas.

(b) (10 points) The Rosseland opacity is an average opacity independent of frequency. If the energy from the interior of a star is transfered by radiation, show how the Rosseland mean opacity is related to the luminosity at a certain radius $L(r)$ and the temperature gradient. The Kramers opacity law is valid in the interior of low-mass (up to solar) stars, for which the opcity is mostly due to energy transition of free electrons (bound-free and free-free). Show that the opacicy $\kappa = \kappa_o \rho T^{-7/2}$, where $\rho$ is the density and $T$ the temperature.

(5) (10 points)

Explain how the spectral type of a star can be determined by observations. The spectral type can be used to estimate *roughly* the effective temperature of a star. What is the definition of the effective temperature. There is however no perfect correspondence between temperature and spectral type. The Sun, a G2 V star, has $T_{\rm eff} \sim 5800$ K. A G2 III giant star on the other hand has $T_{\rm eff} \sim 5400$ K, i.e., lower than that of a main-sequence star of the same spectral type. Use the Saha equation to explain this.

(6) (10 points)

Describe two recent research developments or important results in stellar astronomy or astrophysics. You should briefly outline the main results and try, as best as you can, to explain how they contribute to our understanding of stars.