Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

30th May, 2002

(1) (30 points)

(a) (10 points) Write down the equations that describe the internal structure of stars. State clearly the meaning of each symbol, and the governing physical principle of each equation.

(b) (20 points) Considering a hypothetical one solar-mass star made of pure helium (no hydrogen or metals), compare as quantitatively as possible its physical parameters (e.g., luminosity, temperature structure, main sequence life time, etc) with those of the Sun.

(2) (10 points)

Given the chemical abundances by mass of hydrogen (X), helium (Y), and heavier elements (Z)

(a) (5 points) Show that the equation of state of nondegenerate plasma is $P = \rho k_{\mathrm{B}} T[2X + 3Y/4 + Z/2]/m_{\mathrm{H}}$, where P is the pressure, ρ the gas density, T the temperature, k_{B} the Boltzmann constant, and m_{H} the mass of the proton.

(b) (5 points) What is the equation of state for radiation inside a star?

(3) (10 points)

(a) (5 points) What are the lines of *observational* evidence of star formation in the Galaxy?

(b) (5 points) What are the lines of evidence, direct or indirect, about nuclear reaction processes in the stellar interior?

(4) (30 points)

A chemically homogeneous star has mass M, radius R, and a density profile $\rho = \rho_{\mathrm{c}}(1 - r/R)$.

(a) (10 points) Determine the core density ρ_{c} and core pressure P_{c} in terms of M and R. Sketch the pressure profile.

(b) (10 points) If the star is fully convective ($\gamma = 5/3$) (except its core and its atmosphere), sketch its temperature profile.

(c) (10 points) Assume that the interior of the star is ideal gas and get a temperature profile from the density profile and the pressure profile in (a). Compare this result with the result in (b). Comment on the validity of the linear density profile assumption.

(5) (20 points)

(a) (10 points) For non-relativistic degenerate gas ($P = K_1\rho^{5/3}$) derive the mass-radius relation for a white dwarf.

(b) (10 points) As mass increases, the electrons are pushed closer together and into higher momentum states. Eventually, the electrons become relativistic ($P = K_2\rho^{4/3}$). Show that there is a limiting mass for this case and express the limiting mass in terms of K_2.