PHD QUALIFY EXAMINATION — STELLAR ASTROPHYSICS

September, 2000

(1) (15 points)
In a clear night, a star has an apparent magnitude of $m_{\rm v} = 6.80$ when it is at zenith angle of 30°, and $m_{\rm v} = 6.97$ when it is at zenith angle of 60°. Estimate its $m_{\rm v}$ when it is measured
(a) at the zenith, and
(b) outside the Earth's atmosphere.
(c) What is the optical depth $\tau_{\rm v}$ of the atmosphere at the zenith angle of 45°?

(2) (25 points)
For a pure helium gas with pressure $P_{\rm g} = 10^3$ dyn cm^{-2} $= 10^2$ N m^{-2}, and temperature $T = 12,000$ K. If we ignore the ionization of He$^+$ to make He^{+2}, estimate the number density ratio $n({\rm He}^+)/n({\rm He})$, electron pressure ($P_{\rm e}$) and gas density ($\rho$). Remember $P_{\rm g} = [n({\rm e}^-) + n({\rm He}^+) + n({\rm He})]\, k_{\rm B}T$. Note that $\chi_{\rm ion}({\rm He}) = 24.58$ eV.

(3) (25 points)
(a) (10 points) Show that the equation of state for a completely degenerate and nonrelativistic electron gas may be written as

$$P = K\rho^{5/3}\,,$$

where P and ρ respectively denote the pressure and density of the electron gas and K is a constant.
(b) (10 points) Show also that the equation of state for a completely degenerate and extreme relativistic electron gas is given by

$$P = K'\rho^{4/3}\,,$$

K' is a constant.
(c) (5 points) Discuss the possible roles played by these two equations of state for late stage stellar evolution.

(4) (25 points)
Consider an isothermal gas cloud in low density surroundings and focus our attention on a small volume of gas at its surface. For simplicity we also assume a spherical cloud.
(a) (5 points) Write down the appropriate equation of motion for this volume element,
(b) (15 points) Show that the instability of gravitational contraction (i.e., inward acceleration) may be cast in the form

$$M \geq M_{\rm critical} \approx KT^{3/2}\rho^{-1/2}\,,$$

where M represents the mass of the cloud, T denotes the temperature of the gas, ρ is the density of the cloud and K is a constant.
(c) (5 points) Estimate the limiting mass $M_{\rm critical}$ of different phases of the interstellar medium in our galaxy.

(5) (10 points)

Consider a star in which both radiation pressure $P_{\rm r}$ and gas pressure $P_{\rm g}$ are important. It is then convenient to define the ratio of the gas pressure $P_{\rm g}$ to total pressure P such that

$$P_{\rm g} = \frac{N_{\rm A} k_{\rm B}}{\mu} \rho T = \beta P \,, \qquad P_{\rm r} = \frac{1}{3} a T^4 = (1-\beta) P \,,$$

where $N_{\rm A}$ is the Avogadro's number, $k_{\rm B}$ is Boltzmann's constant, μ denotes the mean molecular weight of the perfect gas, ρ is the gas density and T is the temperature of the gas. Show that the total pressure may be cast in the simple form

$$P = K \rho^{\gamma} \,.$$

Find γ and evaluate the constant K in terms of the constants $N_{\rm A}$, $k_{\rm B}$, μ, a and the ratio β. Discuss the role played by this equation of state.

$a = 4\sigma/c = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}

$c = 3.00 \times 10^{8}$ m s^{-1}

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$h = 6.626 \times 10^{-34}$ J s

$k_{\rm B} = 1.38 \times 10^{-23}$ J K^{-1}

$m_{\rm e} = 9.11 \times 10^{-31}$ kg $= 9.11 \times 10^{-28}$ g

$m_{\rm H} = 1.67 \times 10^{-27}$ kg $= 1.67 \times 10^{-24}$ g

$N_{\rm A} = 6.02 \times 10^{23}$ mol^{-1}

eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg

$L_{\odot} = 3.86 \times 10^{26}$ W

$M_{\odot} = 1.99 \times 10^{30}$ kg

$R_{\odot} = 6.96 \times 10^{8}$ m

$T_{\rm eff\odot} = 5780$ K