PHD QUALIFY EXAMINATION — STELLAR ASTROPHYSICS

March, 2000

(1) (20 points)
A low mass sequence star has mass M, radius R and effective surface temperature $T_{\rm eff}$.
(a) (15 points) Find the average time a photon resides inside the star and the average opacity inside the star in terms of M, R and $T_{\rm eff}$.
(b) (5 points) Estimate the average opacity inside the Sun.

(2) (20 points)
A chemically homogeneous star of solar abundance has mass M and a density profile $\rho = \rho_{\rm c}(1 - r/R)$, where R is the radius of the star.
(a) (5 points) Derive the pressure of a completely degenerated and nonrelativistic electron gas.
(b) (15 points) Find the core pressure of the star. Derive an expression for the minimum mass of the star to ignite thermonuclear fusion of hydrogen.

(3) (20 points)
The opacity and the fusion power density of a main sequence star are given by $\kappa \propto \rho^{\alpha} T^{\beta}$ and $\epsilon \propto \rho^{\gamma} T^{\delta}$.
(a) (10 points) Find the luminosity-effective surface temperature relation of the following stars:
(i) gas pressure is negligible, $\alpha = 0$, $\beta = 0$, $\gamma = 2$ and $\delta = 17$;
(ii) radiation pressure is negligible, $\alpha = 0$, $\beta = 0$, $\gamma = 2$ and $\delta = 17$; and
(iii) radiation pressure is negligible, $\alpha = 1$, $\beta = -7/2$, $\gamma = 2$ and $\delta = 4$.
Don't forget the mean mass of gas particles μ in the last two cases.
(b) (10 points) Find μ if the mass fraction of hydrogen, helium and heavy elements are X_1, X_4 and $X_{\rm A}$ and the gas is fully ionized. (State clear any approximation made.) Suggest the mass range of each of the above three cases. Sketch the three cases for helium stars and stars of solar abundance on an H-R diagram.

(4) (20 points)
Consider thermonuclear fusion in stars.
(a) (10 points) At the core of the Sun, the temperature is 1.5×10^7 K, the density is 10^5 kg m^{-3}, hydrogen mass fraction is 0.5, proton-proton fusion rate per unit volume is 5×10^{13} s^{-1} m^{-3}, and the average energy release per proton-proton fusion is 15 MeV.
(i) Find the mean life time of a proton before it fuses with another proton.
(ii) Estimate the burning core radius.
(b) (10 points) In a burning helium core of a star, the temperature is 2×10^8 K, and the density is 10^8 kg m^{-3}. Find the number density of ^{4}He, ^{8}Be and ^{12}C*. (Note that the mass difference: $m_8 - 2m_4 = 91.8$ KeV/c^2 and $m_{12}^* - 3m_4 = 379.5$ KeV/c^2.)

(5) (20 points)

Stellar evolution can be discussed on an H-R diagram.

(a) (10 points) Locate the main sequence stars, the Hayashi line and the Hayashi forbidden region an H-R diagram (with axes properly labeled)? What is the physics behind the Hayashi line? Discuss the role played by the Hayashi line in the pre- and post-main-sequence evolution of high and low mass stars.

(b) (10 points) Discuss what happen at the locations labeled A, B, C, D, E, F, G, H and K on the post-main-sequence evolutionary track of a five-solar-mass star as shown in the figure.

$a = 4\sigma/c = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}

$c = 3.00 \times 10^{8}$ m s^{-1}

$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}

$h = 6.626 \times 10^{-34}$ J s

$k_{\rm B} = 1.38 \times 10^{-23}$ J K^{-1}

$m_{\rm e} = 9.11 \times 10^{-31}$ kg

$m_{\rm H} = 1.67 \times 10^{-27}$ kg

$N_{\rm A} = 6.02 \times 10^{23}$ mol^{-1}

$L_{\odot} = 3.86 \times 10^{26}$ W

$M_{\odot} = 1.99 \times 10^{30}$ kg

$R_{\odot} = 6.96 \times 10^{8}$ m

$T_{\rm eff\odot} = 5780$ K