PHD QUALIFY EXAMINATION — STELLAR ASTROPHYSICS

September, 1999

(1) (25 points)
Consider a pure hydrogen stellar envelope at $\rho = 10^{-8}$ g cm^{-1} and $T = 20,000$ K.

(a) (5 points) Calculate the relative populations of the three lowest levels of hydrogen atoms.

(b) (10 points) Calculate the mean molecular weight μ.

(c) (10 points) Estimate the partial pressure of the atoms, ions, electrons and radiation.

(2) (25 points)
A homologous contracting star with $\kappa_g = \kappa_o \rho^{0.5} T^{-2.0}$ is in radiative equilibrium.

(a) (15 points) Compute the changes in its central pressure, central temperature, luminosity and effective temperature, when its radius has contracted to 80% of the initial value.

(b) (10 points) Derive the evolutionary track of such contracting pre-main sequence stars in the H-R diagram.

(3) (25 points)
At the end of hydrogen burning, a low mass star has an isothermal helium core, a hydrogen burning shell and a hydrogen-rich envelope.

(a) (5 points) Derive the virial theorem of the isothermal core and give reasons to justify the validity of the theorem.

(b) (15 points) Show that there is an upper limit to the ratio of the mass of the isothermal core to the total mass of the star (i.e., $M_c/M_* \le q_{\rm SC}$, the Schönberg-Chandrasekhar limit). State clearly the approximations you made.

(c) (5 points) What is the approximate value of $q_{\rm SC}$ for a one solar mass star? Is it possible for such a star to evolve to a stage where $M_c/M_* > q_{\rm SC}$? If yes, describe in detail what happens to the core next.

(4) (25 points)
A white dwarf is completely degenerate and has a density profile $\rho = \rho_{\rm c}\big[1 - \alpha(r/R)^{\beta}\big]$ where $0 \le \alpha \le 1$.

(a) (10 points) From the density profile find the central pressure of the star in terms of its total mass M and radius R.

(b) (10 points) If the central number density $n \ll (mc/h)^3$, derive an expression for R in terms of M and other fundamental constants and parameters related to the constituents of the dwarf. Sketch the positions of white dwarfs of different masses on an H-R diagram.

(c) (5 points) Find the maximum mass of the white dwarf in terms of fundamental constants and constituent parameters? Will this maximum mass be different if the dwarf is rotating, and why?

$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$a = 4\sigma/c = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$m_{\rm e} = 9.11 \times 10^{-28}$ g
$m_{\rm H} = 1.67 \times 10^{-24}$ g
eV $= 1.6 \times 10^{-12}$ erg
ionization potential for hydrogen is 13.6 eV